

Inertial Guidance, Navigation, and Control Systems

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Introduction

THE Department of Defense (DoD) and NASA invested more than 1.5 billions of dollars in guidance, control, and navigation during the past fiscal year. The technology represented by such a funding level (which does not even include other major customers such as the Federal Aviation Agency) is a far cry from the fixed rifle sight, the sextant, the eyeball, and "chewing gum on the windshield."

A few clarifying definitions should be stated at the outset. A system that simply indicates the position and velocity of a vehicle is a navigation system. If a navigation system is placed in a closed loop with a vehicle's controls via a guidance computer in order to control the position and velocity of the vehicle, a guidance (and control) system results (Fig. 1).² The related processes of guidance and control may be distinguished as follows: Guidance is the process of continually correcting the velocity vector of the center of mass of the vehicle, so that the vehicle will reach a specified point in space

and time. Control is achieved by movements about the center of mass to adjust the lift and drag forces on the vehicle. (Control of engine thrust may or may not be involved in these processes.) Three commonly used classes of guidance and navigation systems are 1) inertial systems, "without the use of any radiation, either natural or man-made"; 2) radiation systems (that rely upon optical, radio, radar, or infrared data, and that include command systems and homing systems); and 3) externally aided inertial systems (combinations of types 1 and 2).

The relative emphasis being placed on inertial techniques in the United States is indicated by a relative breakdown of DoD and NASA funds for guidance systems and components for fiscal year 1964 (Fig. 2). The DoD and NASA both spent about 50% of their research and development (R&D) money and 80% of their production funds (procurement for operational systems) on inertial systems. (The marked difference between relative R&D and production allocations is due to the high unit production costs of inertial systems relative

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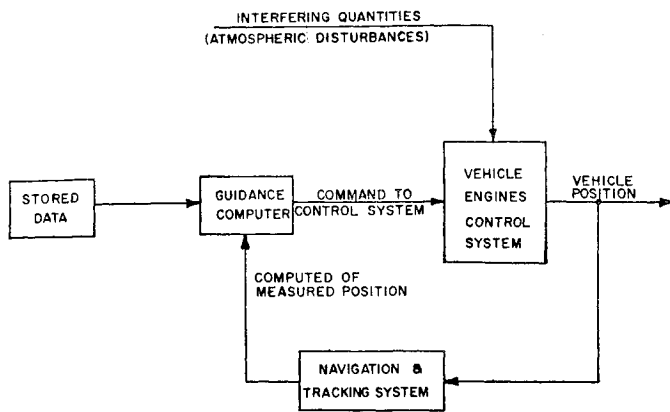


Fig. 1. Generalized guidance and control system.

to radiating systems.) By services, the Army and Navy spent one-fourth and one-half of their total guidance funds on inertial systems, respectively, but the Air Force, with 64% of the DoD guidance budget, spent 87% in the inertial area, so that 75% of all DoD guidance funds went to inertial guidance. For this reason (and because we could not cover adequately all types of guidance, navigation, and control), we shall confine this survey paper to inertial systems and components.

Historical Development of Inertial Guidance

The birth date of inertial guidance is a matter of interpretation, because it was neither the result of any single event nor a discovery attributable to any single individual.²

Sir Isaac Newton* is credited with the law of inertia, but the clue was given earlier by Galileo, who wrote in his *Two New Sciences*:

... any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of acceleration or retardation are removed, a condition which is found only on horizontal planes; for in the case of planes which slope downward, there is already present a cause of acceleration; while on the planes sloping upward, there is retardation; from this it follows that motion along a horizontal plane is perpetual; for, if the velocity be uniform, it cannot be diminished or slackened, much less destroyed.

In 1896, the use of gyroscopes was proposed for marine sextants for those cases where the navigator could see the stars but could not see the horizon. This invention, by Admiral Fleurius³ of the French Navy, consisted of a small top mounted in a vacuum box attached to the sextant; lines ruled on the glass lenses were used to observe the vertical as the top precessed around the vertical. The shipboard gyrocompass, invented in Germany in 1908 by Hermann Anschütz-Kaempfe, elevated the spinning rotor from the role of a toy or mathematical novelty to that of useful instrumentation. In 1911, Elmer A. Sperry obtained his first U. S. patent on a gyrocompass. On July 15, 1924, U. S. patent 1501886 was issued to C. G. Abbot for a system with a three-axis gyro and a gravity pendulum, the first system, as far as we know, with some inertial capability.

An inertial guidance system operates in a coordinate system that is not rotating or accelerating with respect to the "fixed stars." An inertial or Newtonian coordinate system is one in which Newton's laws are valid. This "inside-out"-type of definition bothered Einstein,⁵ who asked:

Can we formulate physical laws so that they are valid for all coordinate systems, not only those moving uniformly, but also

* Slater⁴ suggested that "inertial guidance" should be called "Newtonian Guidance" because of the law of gravitation and the laws of motion of equal significance to its functioning, but "inertial guidance" is likely to stay with us.

those moving quite arbitrarily, relative to each other? If this can be done, our difficulties will be over. We shall then be able to apply the laws of nature to any coordinate system.

He solved the problem with his general relativity theory, which, when applied to inertial coordinate systems, gives the special relativity theory as a special case.

At least three coordinate systems (or their equivalents) must be considered in any mechanization of inertial guidance system.^{6,7} Nongravitational specific force measurements are made in inertial coordinates. The aircraft or missile guidance system must mechanize or remember coordinates rotating with the earth; this is concomitant to saying that the system must carry a clock since, for all practical purposes, the earth's angular velocity vector is constant. The vehicle's coordinates must be known, because guidance commands are implemented in these coordinates through thrust vector control or by deflecting aerodynamic surfaces.

Schuler Tuning and the Stable Platforms

The Anschütz and Sperry gyrocompasses were gyroscope-pendulum combinations and suffered from lack of accuracy during turns. Anschütz was greatly troubled in 1906 as he was working on the development of the gyrocompass by a paper published by O. Martienssen⁸ which showed the gyrocompass would have very large errors under north-south acceleration. He contracted a study by Maximilian Schuler, who determined that the gyrocompass reading would be insensitive to applied accelerations if its pendulous element had a natural period of oscillation of 84 min in the earth's gravity field. This condition, called "Schuler tuning,"⁹ immediately improved the gyrocompass art; however, Schuler¹⁰ reported in his first paper in 1923 that, insofar as he knew, no one had succeeded in building a gyropendulum as a vertical indicator with a period longer than 30 min. Schuler's paper is paraphrased and interpreted by Wrigley⁹ and translated in its entirety from the German by John M. Slater in Appendix A of Ref. 11. Today, Schuler tuning is a common characteristic of most inertial guidance systems.

A remarkable man who contributed to the development of Tragheitsortung (inertial guidance) was Johann Maria Boykow, a German actor and naval officer who conceived a stable platform similar to that used in many inertial navigation systems today. Boykow's U. S. patent is dated February 22, 1938. Among his numerous conceptions is a platform stabilized with three single-degree-of-freedom gyros and carrying two accelerometers. Velocity information is fed back to torque the two horizontal gyros in order to maintain local coordinates.

Engineering Research and Production

Following the invention of the gyrocompass, literature on gyroscopes began to emphasize purposeful applications and the improvement of performance characteristics (a comprehensive listing of papers through 1941 is given in Ref. 12). Sperry Gyroscope Company was the first company started in the United States for the purpose of manufacturing gyroscopic instruments for marine and aircraft applications. The Arma Division of American Bosch Arma Company started making floated gyros in the early 1920's. (Note that Anschütz relied on liquid flotation much earlier.) Norden entered the gyro field about 1930; Norden's systems work included bombing, navigation, fire control, missile guidance, reconnaissance, terrain warning, and clearance. Norden also produced the AN/ASB-1 and AN/ASB-7 bombing equipment used extensively in Navy heavy attack aircraft.

The first practical inertial guidance system was developed by the German Peenemünde group for the V-2 rocket.² This system consisted of a gyro assembly with a clock-driven pitch programmer and an inertial coordinate system to control the missile's attitude. One pendulous integrating gyroaccelerometer was mounted along the thrust axis to measure velocity

and to give a shutoff to the engine when the required velocity was achieved. This system introduced gravity as a bias in open loop fashion. Among those who made substantial contributions to the success of the V-2 guidance system were Walter Hausermann of the NASA Marshall Space Flight Center, Helmut Schlitt of Bell Aerosystems Company, and Theodore A. Buchold of General Electric.

The V-2 inertial guidance system, primitive by today's standards, was the only operable system in the world until prototype aircraft inertial navigation systems developed at Massachusetts Institute of Technology (MIT), North American Aviation, Northrup Aviation, and Hughes Aircraft Company were flight tested. The MIT system, called FEBE, was flight tested in a B-29 flown from Massachusetts to New Mexico in the spring of 1949. The Hughes system was tested in a B-26 flown from Lake Muroc to Sacramento. The Hughes system consisted of Sperry A-12 autopilot gyros (drift rate about 0.5 deg/min) mounted in a platform supported by a gimbal system that was about 3 ft in diameter. Two stars were monitored by a telescope capable of night stellar tracking in order to compensate for the very large gyro drift rates. Continuous navigation accuracy better than 6 miles was recorded.

John M. Slater, in his introduction to Ref. 11, said:

As of early 1946, it is not apparent that even the principles of inertial navigation were clearly understood or defined. On the one hand, efforts were persisted in to make use of gravity vertical references under hopelessly unsuitable conditions; on the other, in many proposals to make use of accelerometers and integrators, there was a lack of understanding of the principles of feedback for gravity compensations.

The inertial navigation system in its modern form, including gyro-stabilized accelerometers, integrators, and computers for gravitational acceleration was apparently evolved rather than invented. . .

A number of educational institutions are carrying out research and educational activity in inertial guidance, including MIT, University of Southern California at Los Angeles, California Institute of Technology, Polytechnic Institute of Brooklyn, Stanford, Cornell, Case Institute of Technology, University of Virginia, University of Michigan, City College of New York, University of Minnesota, and Ohio State University. Special recognition is due C. Stark Draper of MIT, who recognized the possibilities of inertial guidance in the 1930's and worked incessantly toward the fulfillment of these early dreams¹³:

MIT Instrumentation Laboratory work on inertial guidance was a direct continuation of studies in aircraft instruments started in 1930. These studies were largely concerned with . . . application of inertial reference to the operating problems of aircraft and naval vessels. [At the] end of 1944 . . . discussions with members of the Armament Laboratory of the Wright Air Development Center led to the initiation of a project directed toward the development of non-radiating bombsights for aircraft. Details were necessarily vague but the great improvements to be made in gyro units, accelerometers, servodrives, amplifiers, time drives, etc., were recognized. All of the essential problems were attacked at the same time, with the clear realization that several years of continuous and coordinated effort would be needed before the possibilities and limitations of inertial-guidance systems could be established. . .

Many industrial organizations are responsible for the stature to which inertial instruments and systems have risen. The Arma Division of American Bosch Arma has the distinction of building the Atlas missile inertial system, the first operational all-inertial ICBM in the United States. (Some other companies have already been cited in this paper, but equally important ones have not been cited.) The current competition in this field is good in many ways, but it also presents the customer with many difficulties in choosing components, as noted by Farrior.¹⁴

Inertial Components

Inertial sensors are used to obtain information on orientation and accelerations relative to a reference frame in which

Newton's laws are valid. Orientation (attitude) measurements, obtained by gyros, provide angular information. Acceleration measurements, integrated twice for distance, provide linear information.^{15, 16} Missiles and space vehicles operate in three dimensions; hence three independent angular and linear measurements must be instrumented. Cruise vehicles (aircraft, ships, submarines) operate essentially in a two-dimensional plane (the earth's surface), and hence require instrumentation for three angular measurements and only two linear measurements. Altitude measurements in cruise systems are normally not sensed by inertial techniques. The altitude channel, which is parallel to the gravity vector, is unstable in an inertial navigation system for cruise vehicles. In an orbital system, where gravity is balanced by centrifugal force and the "free fall" or zero g exists, the instability in the altitude channel of an inertial system disappears; the unstable axis rotates 90° and now shows up as the range channel.^{11, 17}

Gyroscopes

Orientation relative to inertial space requires some physical factor that can be geometrically isolated from its surroundings. The only factor that has fulfilled this requirement in a practical engineering sense to date is the angular momentum of a spinning body. Only three laboratory concepts among the unconventional angular sensors discussed later do not use the concept of angular momentum; all other angular sensors are merely different ways of isolating spin momentum from its environment. Any physical quantity having directional characteristics that can be maintained in the face of interferences can be used for spatial orientation reference. To the best of our knowledge, no radically new concepts of an unclassified nature have appeared during the past year which cannot be classed fundamentally as spinning rigid bodies, vibratory momentum devices, nucleon devices, or closed-path radiation devices.

The two primary properties of gyros which are utilized in navigation systems are the following.

1) Pointing to a fixed direction in space. As applied to a rotating body, Newton's first law says that a rotating body will continue with its present angular velocity about a fixed axis of rotation until action upon some external torque.^{11, 18, 19} A practical gyro instrument, however, drifts because of unwanted torques resulting from less than perfect machining, bearing and lubrication frictional torques, slight changes in material properties causing mass unbalances, dust and other foreign matter causing undesirable torques, material impurities and imperfections, dimensional instability of machine parts, and degassing.²⁰

2) Ability to convert an angular velocity to a torque, and vice-versa. The spin vector precesses toward the torque

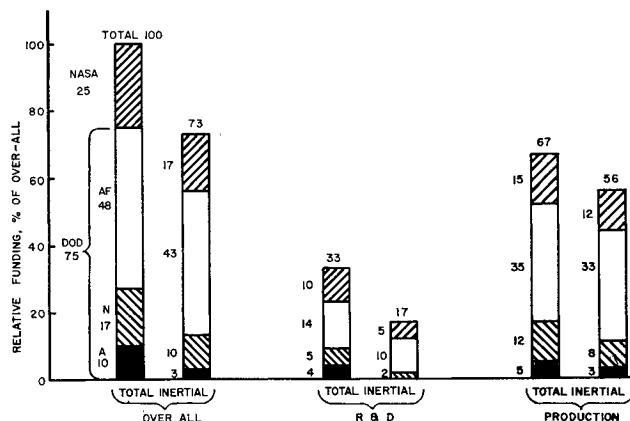


Fig. 2 Relative expenditures for guidance systems and components for fiscal year 1964. AF = Air Force; N = Navy; and A = Army.

vector. Understanding the operation of a gyro is therefore a three-dimensional problem. This gyroscopic property is reversible, i.e., a torque input results in an angular velocity output (precession), and forced precession results in a torque output.²¹

Gyros are naturally classified into two main categories depending upon which of these two primary properties are predominant in the instrumentation (Fig. 3). The two-degree-of-freedom (2DF) is sometimes called a "free" or "amount" gyro; it uses the property of gyroscopic rigidity in space and can be used to measure directly a vehicle's angular deviation from any given reference coordinate system. The interaction of torque, spin, and precession is the primary property instrumented in a single-degree-of-freedom (SDF). An SDF may be either a "rate" (angular velocity output) or a "rate-integrating" (angular displacement output) gyro; the former has a spring restraint (torque proportional to displacement) to counteract output gyroscopic torques (see Fig. 3), whereas the latter has a viscous restraint (torque proportional to velocity). Some 2DF gyros are also instrumented as "rate" gyros. An inertial navigation system requires either two 2DF or three SDF gyros in order to establish inertial coordinates in three dimensions. The "quality" of this inertial reference depends on the precision of the gyro instruments.

The floated integrating gyro²² with electromagnetic centering is the most accurate unit available today for operations involving thrust and gravity. Random drift characteristics²³ of production gyros vary a great deal, depending on what the customer is willing to pay. If the customer requires very high performance, the percentage of assembled gyros which pass acceptance tests may be very low, and the unit cost of acceptable gyros may be measured in tens of thousands of dollars. Reasonable progress has been made during the past year in improving the inherent accuracy of these instruments and in obtaining this accuracy with a higher fraction of units produced, for longer periods of time, and at less cost and weight. New materials, such as ceramic rotors, are being introduced because of certain favorable material properties. Case rotation to reduce mass unbalance drifts is being practiced on a wider scale. Modulation of angular momentum to separate error sources and thus permit better compensation has been demonstrated in a number of gyro laboratories. The long debate over gas bearings vs lubricated ball bearings

continues; progress is being made on both fronts.²⁴ The fundamental physics of lubrication, long a speculative subject, is now beginning to be understood. The race between gas bearings²⁵ and ball bearings is still competitive; there are merits to each type depending on the specific requirements of the system being instrumented.

A practice that is gaining favor in some applications such as ship inertial navigation systems (SINS) and some ballistic missile platforms is the use of a redundant gyro to calibrate the basic platform gyros. The redundant gyro is sometimes the same type of instrument that exists in the platform, but case rotation and other techniques are utilized to improve its performance.

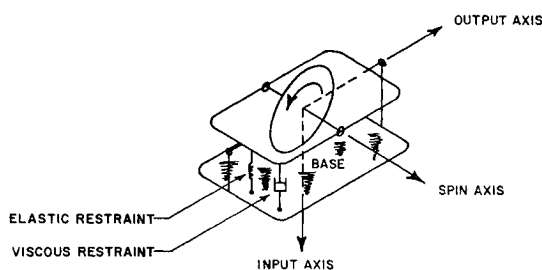
A significant advance in gyro state-of-the-art is digital pulsed torquing. This advance, coupled with the great advances in computer technology, has made strap-down inertial systems (no gimbals, body-mounted gyros and accelerometers) competitive in performance with conventional gimballed systems,^{17, 26} for some applications where vehicle angular rates are not excessive.

Spinning mass gyros: free-rotor types

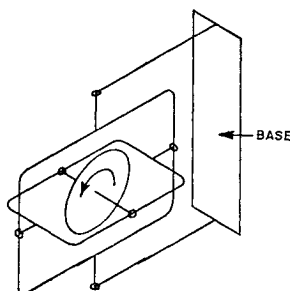
In principle, the free-rotor gyro (Fig. 4) operates in the same way as the gimballed gyro. Its rotor, which is spherical, may be supported by gas bearings, electrostatic forces, or magnetic forces.²⁷ The rotor generally has an electrically conductive rim that is driven as an eddy current motor by a case-mounted stator. For a gas-bearing-supported rotor, the gap between the rotor and case is filled with gas under pressure. The gap in the electrostatic-supported gyro is in a vacuum, and the rotor is kept centered by electrostatic forces. The motor is used to start the gyro and obtain a desired angular momentum; the rotor then coasts for months, since friction torques in the vacuum are extremely small.

A very interesting, although not necessarily the most practical, electromagnetically supported gyro is the cryogenic gyro.²⁷ In this instance, the rotor is a superconducting niobium sphere immersed in liquid helium. Magnetic lines of force do not penetrate it. The supercurrents flowing at the surface of the sphere interact with the external magnetic field to provide the supporting force as well as affording stability without a separate servo control. The rotor is suspended within a segmented bearing assembly, which is also made of niobium. The magnetic flux, which flows in the gap between the rotor and the bearing segments, is produced by two circular bearing coils wound with niobium wire surrounding an evacuated cylindrical enclosure housing the gyro. Normal excitation current of 1 amp produces a flux density at the rotor surface of approximately 1000 gauss. Continuous current flow has been maintained during gyro operation from an external supply, but a persistent superconducting current could also be used. Two torquers located in the center of the rotor act as magnetic bearings to maintain spin axis alignment to the gyro case during rotor spin up. Motor drive torques are produced by a superconducting stator also mounted in the center bore of the rotor. Because of the limited angular freedom (1.5°) between the gyro rotor and the torque-motor assembly, a cryogenic gyro of this type would be mounted on a stabilized platform that is servo driven to follow the rotor spin axis. The principle error sources are asphericity and erroneous center-of-gravity location of the rotating sphere, trapped stray magnetic fields, and energy transfer due to readout devices. A severe difficulty that remains to be solved is a.c. loss in the superconductor. The General Electric Company and the Jet Propulsion Laboratory are the two primary centers of cryogenic gyro research.

Another interesting free-rotor gyro uses a rotating fluid sphere²⁷ instead of a metallic ball. It does not appear to require the degree of sophistication in manufacturing that conventional gyros of the same characteristics require. A spinning mass of high-density, low-viscosity fluid (rather than



a) Single-degree-of-freedom gyro



b) Gimballed two-degree-of-freedom gyro

Fig. 3. Gimbaled-type gyros.

the conventional heavy wheel) provides the angular momentum. Viscous shear at the boundary of the fluid sphere causes it to rotate at the same speed as the cavity; in the absence of inputs, the pressure distribution at the cavity boundary is symmetrical along the spin axis. If a rate with a component orthogonal to the spin axis is introduced, the angular momentum of the fluid sphere will cause it to try to remain fixed in space, and a small angular separation will exist momentarily between the cavity spin axis and the fluid-sphere spin axis, creating an asymmetric pressure distribution. Two ports in the cavity channel the fluid to a transducer, which measures the pressure differential with respect to the cavity spin axis. Since the ports are rotating in space with the cavity, and the angular difference between the fluid and cavity spin axis is fixed in space or a sufficient time interval, the pressure differential measured will vary sinusoidally, with a period equal to the period of the cavity rotation and an amplitude proportional to the magnitude of the input rate. Now, by introducing appropriate phase-sensitive demodulation with respect to a voltage obtained from a reference generator on the gyro rotor or cavity, components of the input rate can be found in two axes. Since the viscous shear at the cavity boundary will eventually return the fluid body to its original position colinear with the cavity axis, a step function angular input will lead to an instantaneous angular displacement that will decay to zero in an exponential manner.

Gyros used in space systems should be designed specifically for the space environment. The zero- g condition, coupled with the requirements for low power consumption and long times of operation, result in somewhat different design criteria than utilized in gyros used near the earth's surface. It is possible that some of the unconventional gyros may be useful in space applications.

The *electrostatic gyro* (ESG) has demonstrated performance on a par with the best of any other type of gyro. This gyro consists of a hollow spherical beryllium rotor suspended in a hard vacuum by strong electrical fields.²⁸ Credit for conceiving the ESG is due A. T. Nordsieck, then of the University of Illinois. The primary sources of drift in the ESG are mass unbalance of the rotor, magnetic torques resulting from interaction on induced currents in the rotor with the exciting magnetic field, and electrical torques due to geometric imperfections in the suspending field and the rotor. It is well known in electrostatics that the suspension by fields of a body with constant charge distribution is not stable in all directions. In the ESG, fields are created by placing high voltages across electrodes arranged concentrically with the rotor. These voltages are carefully controlled by servo techniques to force the rotor continuously toward the center of the gyro case. The rotor never comes in contact with the case. The ESG requires continuous use of a high-performance servo; otherwise, it should be ideally suited for space applications, provided that the techniques in internal compensation are fully developed. Honeywell and General Electric Company are industrial concerns performing government-funded research on it.

Vibrating momentum gyros

In the vibratory momentum gyro, first described as the tuning-fork gyro in 1953 by Sperry engineers, the linear momentum of an vibrating mass is the analog of the angular momentum of a spinning rigid body. Linear momentum can be used as an inertial characteristic for measuring angular motions. Gulton Industries investigated the use of a radially vibrating ceramic disk as an angular rate sensor. The difficulty with this concept was isolating the driving frequency from the pickoff frequency. Westinghouse uses a ceramic cylinder; under an oscillating excitation voltage, the moment of inertia about the longitudinal axis oscillates. The piezoelectric cylinder acts as a crystal to control the frequency of

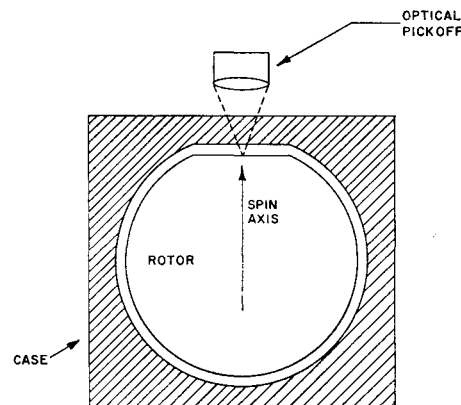


Fig. 4 Free-rotor, two-degree-of-freedom gyro.

the driving oscillator. The primary difficulty with this device is null stability.²⁸ No significant progress has been made in recent years.

Nuclear gyros

Nucleons are much better magnetometers than gyros. As solid-state gyros, they might appear to be useful. Although progress has been made in the last year, it appears that the practical use of nucleons must await many years of laboratory research. The nuclei of certain atoms exhibit an angular momentum and a magnetic moment. Their angular momenta are normally randomly oriented in space, but they can be statistically oriented by the action of an external magnetic field. The resultant microscopic angular momentum of a nucleus is stable with respect to inertial space. Nuclear magnetic moments can be observed by means of magnetic resonant effects and atomic beams. Various methods using these techniques have been considered for mechanizing nuclear gyros.^{29, 30} Basically, angular rotations of the case are measured with respect to an ensemble of nuclei aligned in inertial space.

Laser gyros

The laser gyro measures the transit-time difference of two beams of radiation which are propagating in opposite directions around the same closed path.^{27, 28} This is an application of the lesser known of Michelson's two famous experiments with light. The laser furnishes adequate energy levels with coherent wave patterns to permit reasonably accurate and sensitive levels of rate measurements. There remains much research ahead before these instruments will be of practical use.

Accelerometers

All acceleration-measuring devices in use today employ the inertia reaction effect of a proof mass, in some cases restrained to a null position and in others absorbed by a counter reaction. No accelerometers of radically new concept have appeared during the past year. Inertial velocity-measuring devices, as such, are nonexistent because unique inertially referred velocity is meaningless.⁵ Velocity data obtained by measuring the motion of the surrounding medium, such as by aircraft pitot tubes, have been used for many years. Velocity measurement by electromagnetic radiation methods, such as Doppler radar in aircraft^{31, 32} and Doppler sonar in ships, is constantly being improved, as evidenced by the accuracy with which earth-guided space probes have been deployed. Velocity measurement relative to the earth's electric or magnetic fields is a theoretical possibility, but the variations in these fields make this impractical for the accuracies desired and/or required. In most current models of accelerometers, the proof mass is manifested as pendulous unbalance, and

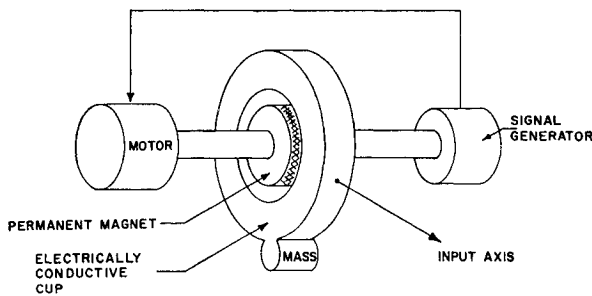


Fig. 5 Magnetic drag-cup velocity meter.

generally is supported by flotation. It is a well-known fact from Einstein's principle of equivalence that an accelerometer cannot separate gravitational acceleration from inertial acceleration; hence the former is computed and subtracted from the measured acceleration in the feedback loop, the process known as Schuler tuning.

Pendulous accelerometers

The null-reading, torque-balance pendulous accelerometer is very common. It may have either analog (steady) or digital (pulsed) measurement data. Such units can be made quite simple and relatively inexpensive and small while providing reasonable accuracy. Pulsed torquing also permits the inherent direct integration required to give velocity information in digital form. This type of accelerometer provides adequate performance for most requirements.

Magnetic drag-cup velocity meter

A sketch of the principle of operation of this accelerometer is shown in Fig. 5. The permanent magnet and the electrically conductive cup are closely spaced coaxial cylinders. The sensing element is a simple pendulum, which deflects when subjected to accelerations; the signal generator sends an electrical signal to the motor which is proportional to deflection of the pendulum. Other electromagnetic methods may be used instead of the signal generator to sense rotations of the pendulum. The motor rotates the permanent magnet in such a direction as to torque the pendulum back to "null" through electromagnetic coupling of the drag-cup and permanent magnet. The drag torque induced in the conductive cup is proportional to angular velocity; hence the electromagnetic coupling acts as a Newtonian fluid (stress is a linear function of shear). This type of accelerometer is unique in the sense that it effectively uses a differentiating element (the drag-cup coupling) in the feedback loop to achieve integration. Motor speed is proportional to velocity, and hence the name velocity meter. This instrument is very accu-

rate and is in large-scale production for ballistic missile guidance systems.

Pendulous integrating gyro accelerometer (PIGA)

The PIGA (Fig. 6) is the most accurate accelerometer in production today. Its use is desirable when accurate velocity measurements are essential, such as in ballistic missiles and space booster operations, even though it is expensive. An integrating gyro precisely measures the torque generated by the pendulum when it is accelerated; the pendulous mass lags as if it were a simple pendulum, free to rotate about the output axis. As the output axis rotates, the signal generator generates an electrical signal proportional to the angle of rotation. This signal is amplified and drives an electric motor that rotates the gyro gimbal about the input axis at a rate proportional to the applied acceleration. The total angle of rotation θ is proportional to the first integral of acceleration, i.e., velocity. The rotation of the motor shaft forcefully precesses the gyro about the input axis and causes it to generate a torque about the output axis in such a direction as to balance the pendulous torque generated under the applied acceleration.

Vibrating string accelerometer (VSA)

The natural frequency of a taut string is proportional to the square root of the tension; therefore it increases if a lagging mass tends to stretch the string, or vice versa. In a practical accelerometer, the vibration string is actually a metal tape that is constrained to vibrating in one plane in order to improve measurement accuracy. Two tapes are attached to a proof mass located between the strings (see Fig. 7). The mass is constrained to moving in one dimension only, the "input" or "sensitive" axis. The metal tapes are placed in a magnetic field and will vibrate when a current runs through them. The vibration is maintained at a constant amplitude and at the natural frequency of the strings by servo-feedback techniques. An acceleration in the direction of the input axis increases the tension in one tape and decreases it in the other. The resulting difference frequency is proportional to the applied acceleration. The sum of the two frequencies is held constant by a tension adjustment mechanism to improve the accuracy and linearity of the instrument. The integral of difference frequency, the number of difference cycles (measured as pulses), is a direct measure of velocity. The VSA can therefore be looked upon as an integrating accelerometer. It is in large-scale use in this country. Since its output is a series of pulses, it is a natural instrument for use with digital systems.

Vibrating quartz accelerometer

This accelerometer is analogous to the VSA. Resiliently supported, mass-loaded capacitor plates are used in conjunction with an inductor or resistor to form an acceleration-responsive oscillator. Since no driving magnets are required, this unit may be simpler and cheaper than a VSA.

Mossbauer accelerometer

An accelerometer utilizing the Mossbauer effect is the nuclear analog of the VSA. The extreme accuracy inherent here has permitted experimental proof of the gravitation-time relationship of general relativity. However, its very sensitivity makes its practical vehicle-borne use questionable; it certainly requires considerably more laboratory research.

Particle accelerometer

The goal of the inertial sensor designer is to create a device that is free of all effects except the physical effect to be measured. The sensitive element of an inertial sensor is sus-

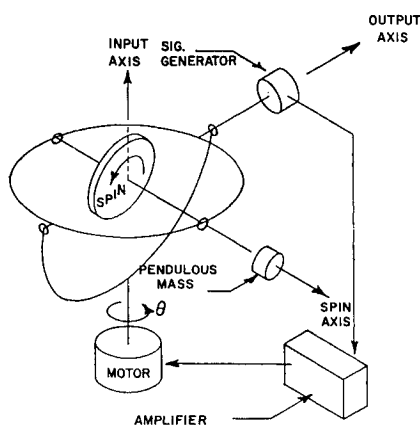


Fig. 6. Pendulous integrating gyro accelerometer (PIGA).

pended by a low-loss suspension in a low-loss medium. The low-loss suspension is perhaps the most difficult to obtain; electric force appears to be the most nearly ideal suspension technique, because it works only on the surface of the suspended mass, so that losses are confined to the dielectric and are small or nonexistent. Electrostatic suspension of large masses would require extremely high electric gradients in an ultra-high vacuum, but it is theoretically possible to use a charged particle as the proof mass. The charge/mass ratio of the particle should be chosen as high as possible; hence the surface/volume ratio should be high, which in turn means that the radius should be as small as is practical. The charged particle reference device is directly applicable as an instrument for sensing accelerations in that they are capable of sensing accelerations along three axes simultaneously.¹⁶ Some development is currently being funded by the government on these particle devices. However, a great deal of laboratory investigation must be conducted before this approach reaches the practical stage.

Other accelerometers

As an indication of the large numbers of different types of accelerometers which have been examined or are currently being studied at various companies and laboratories, Table 1 is representative.

Computers

The past year has brought about very significant progress in computer engineering because microelectronics became an engineering reality.³³ One of the largest and most significant of the microelectronics programs is the Minuteman guidance system development program. Others are the Bureau of Weapons AN/ASA-44 aircraft inertial navigation program, the Bu-Weps Loran C navigation receiver, the AN/ASA-27 airborne digital computer for the Grumman E-2A aircraft, and the guidance computer for the MMRBM weapons system.

Microelectronic circuits permit us "to eat our cake and have it too." Weight and power loads are significantly reduced, computational speed and reliability are greatly increased, and heat dissipation requirements are reduced (environmental cooling equipment can be removed or reduced). In order to realize the greatest economy and quality control in production of microcircuits, it is important to minimize the number of different logic circuits. The best of currently designed computers require 12 to 20 different types of microcircuits. Some microcircuits are easier to produce than others, and some companies seem to have more production success than others. In the past year, some of the problems that limited the production of microcircuits were solved, and unit costs were nearly cut in half. It seems safe to predict that microelectronics will become commonplace in the near future, not only in computers but in platform electronics, sensor electronics, and communications circuits at the cost of equivalent systems built from conventional solid-state circuit elements. Reliability will continue to improve as part counts are reduced.

Conscientious efforts are under way in a number of organizations to advance the state-of-the-art in fabrication of memory elements and in the hardware of interconnecting microelectronics wafers. Memory technology includes thin films, magnetic ferries, metallics, semiconductor memories, electromechanical memories, and core-rope memory.

There is a tendency on the part of guidance system designers to use digital circuits in too many places. Some savings in system cost, complexity, and weight can be realized in some applications at little penalty in accuracy if analog or DDA systems are used instead of digital arithmetic computers. Unnecessary sophistication should be avoided even if sophisticated tools are available.

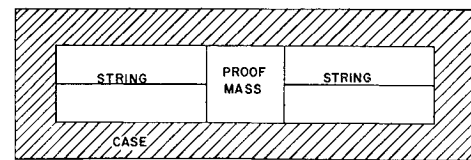


Fig. 7 Vibrating string accelerometer (VSA).

Advances in computer technology have brought about changes in systems technology.³⁴ There is an increasing tendency in advanced ballistic missile guidance systems and space probes to use explicit guidance equations because of the greater targeting flexibility that they provide compared to the use of perturbation techniques. Explicit equations are particularly attractive when external data are used to update the trajectory; examples include the use of azimuth or position data obtained from star sensors and position data obtained by a Doppler radar system.

In some advanced systems under development, comprehensive tradeoff studies have been conducted to determine the degree of centralization of data processing desired. Cost, weight, reliability, and operational mission factors are often of paramount concern in deciding whether or not one central data computer is desired rather than a number of different smaller computers associated with the various subsystems that go together to make up the over-all system. In some instances, the system can be viewed as an information processing problem, in which case the computer becomes the heart and core of the system. As microelectronics become more common, this trend is likely to grow.

Computers with high computational rates and large capacity have now made strap-down inertial systems an engineering reality. In these systems, the physically stabilized platform is replaced by a mathematical platform; many processes carried out physically in normal systems are performed mathematically, thus placing a much greater load on the computer and the initial sensors.

Guidance Systems

The trend continues toward smaller, more accurate, more flexible guidance systems. There have been during the past year some significant changes in emphasis from that of previous years, however. These changes include emphasis on the capabilities for low cost production of aircraft and short range

Table 1 Types of accelerometers

Accelerometer type	Organization
Electrostatics	Bell Aerosystems
Pressure differential	Case Institute
Piezoelectric	Gulton
Liquid column	QMF and CIAF
Seismic mass	Research, Inc.; Kearfott; Whittaker
Semiconductor	Diamond Ordnance Fuze Lab.
Radioactive	Frankford Arsenal
Piezoresistive	Frankford Arsenal
Chatter piezoelectric	Bureau of Standards
Lenz Law pendulous	Donner Scientific Co.
Vacuum tube	MIT, Calidyne, RCA
Polar coordinate	White-Rodgers
Capacitive	Lockheed Missiles and Space
Statham	Naval Material Center
Contact	Royal Aeronautical Establishment, Farnborough
Barium titanate	Bureau of Standards
Ramberg vacuum-tube	Bureau of Standards
Strain	Autonetics
Variable reluctance	Gulton
Differential transformer	Gulton
Bender seismic system	Gulton
Photocell miniature	Jet Propulsion Lab.
Manometer spider tube	Nortronics

Table 2 Organizations responsible for guidance programs

Program	Organization
Atlas E and F	Arma
Titan II and III	MIT and AC Spark Plug
Polaris A1, A2, and A3	MIT and General Electric
Minuteman	Autonetics
MMRBM	General Precision Aerospace
Skybolt (canceled)	Nortronics
Jupiter	Army Huntsville, Ford Instruments and Bendix Eclipse-Pioneer
Pershing	Army Huntsville, Ford Instruments and Bendix Eclipse-Pioneer
Thor	MIT and AC Spark Plug

missile guidance systems, high- g capabilities for re-entry guidance³⁵ and for guiding high-acceleration antimissile missiles, and integrated space guidance system design to include in one basic guidance system the capability for performing boost, in-orbit, and re-entry guidance functions.

Aircraft

Aircraft inertial navigation systems may be divided into the following three categories according to the gyro reference coordinate system used: 1) inertially fixed system, 2) earth-fixed system, and 3) local-vertical-fixed system.

These systems may also be categorized in many other ways: 1) disposition of sensitive axes of the accelerometers relative to the gyro reference axes; 2) three-gimbaled or four-gimbaled systems (the latter are often used to reduce programming requirements or maneuver limitation for preventing gimbal lock); 3) use of three SDF gyros or two 2DF gyros; 4) use of either analog or digital computer; the latter, either arithmetic or digital differential, is required in systems of very high accuracy; 5) whether or not externally measured velocity information is used as an input (e.g., Doppler radar is often used to damp the inertial system; an aircraft carrying a long range air-to-surface missile often uses Doppler velocity after smoothing it, as inertial or launch velocity for the missile system); and 6) whether or not position information derived external to the inertial system is used as an input; such information is generally of the form of angular measurements of stars, or electromagnetic radiation data such as measured by Loran C. Continuous tracking of stars makes possible gyro drift compensation in flight, as well as providing a means for obtaining a stellar fix; however, the vertical and angular measurement capabilities of the star sensors limit the performance capability of the stellar system.

Operational reaction times influence greatly the accuracy of initial alignment in vertical and azimuth. Reducing reaction times is still one of the major difficulties with aircraft inertial navigation systems. Obtaining accurate launch vertical, azimuth, position, and velocity is of particular difficulty in carrier-based aircraft, because these data must be transferred from some external source, such as SINS. The master navigational source itself has errors, and additional errors are introduced in the process of transferring the information. Considerable progress has been made in the past year in improving transfer of angular information in carrier-based systems.

A natural way around the problem of reaction time is to use stellar-Doppler inertial aircraft systems. Initial conditions can be effectively set into the inertial system after take-off by using stellar-derived data. These systems have larger part counts; hence the cost is high and reliability is not as good as in pure inertial systems. Additionally, weather influences the operation of the system. We still have difficulty seeing stars accurately at low altitudes in the daytime, particularly in high haze or humidity conditions, even when the sky is clear. Obviously clouds interfere with the operation of such systems.

Stellar sensors for use in aircraft and missile systems generally are one of three basic types: photomultiplier tubes, vidicon tubes, or solid state sensors. Much progress has been made in the last year in both of the latter two types of sensors. It seems safe to predict that eventually most sensors used in aircraft navigation systems will be of the solid-state type. There is room for much improvement in this area, however, in sensitivity, accuracy, cost, reliability, and maintainability.

Probably the most significant trend in aircraft navigation systems which has only recently developed real momentum is the emphasis on system cost in large production quantities. The Air Force, with the blessing of the Secretary of Defense, has established as a goal a unit cost of \$25,000 per system in large lots for a 1.0 naut mile/hr system. Such a goal is an ambitious one, but it appears to be attainable. However, it will be difficult to achieve such a cost goal using conventional floated gyros; hence, it is possible that the low cost requirements may be the motivating force leading to application of unconventional sensors.

Along with the need to reduce production costs is the need to integrate the navigation systems more fully with the flight control and instrumentation systems. There is a tendency today to develop completely separate flight data systems, autopilot systems, and navigation systems as well as fire control systems for those aircraft equipped with weapons. Obviously many of the measurements required of each of these systems could be made with the same sensors. Much more use, over and above ground position data, can be made from the data gathered by the inertial sensors and the stable platform.

One significant milestone of the past year was the successful test by the Federal Aviation Agency of inertial navigators in many trans-Atlantic flights. Another was flight test measurements of accelerometer and velocity matching techniques for transferring information from a master navigator in an aircraft to slave guidance systems of missiles carried aboard the aircraft.

Although aircraft inertial navigators have been tested in one form or another since 1949, it was not until the 1960's that systems were produced in any significant quantities. Today, we have over 100 operational military aircraft equipped with inertial navigators. Anyone who has flown any of these aircraft knows first hand the great value of precision on board position determination, i.e., value from the standpoints of both military usefulness and safety of flight. We believe that one is safe in predicting a much larger percentage of the aircraft inventory, military as well as commercial, will be equipped with inertial navigators in the future.

Most of the operational aircraft systems in the field today were built by Litton Systems, Inc. These include the F-104, W2F, P3V, and A2F aircraft. Autonetics developed the stellar-Doppler-inertial system for the B-70. Sperry had systems responsibility for the bomb-navigation system in the B-58; this system is a Doppler inertial system monitored by radar and an astrotracker. Norden designed the ASB-7 system for Navy heavy attack aircraft; this is a Doppler system that also incorporates a stable platform. Autonetics has bombing-navigation system responsibilities for the A5A Vigilante aircraft.

Ships and Submarines

Ships inertial navigation systems (SINS) presents the most severe gyro accuracy requirements of all inertial navigation and guidance systems.³⁶ SINS has a big advantage over aircraft, missile, and space systems, however, in that size, weight, and power considerations are much less constraining. Gyro performance in SINS is generally one or two orders of magnitude better than gyro performance in missile systems. The relation between gyro drift and navigational accuracy is very straightforward: 1 min of arc corre-

sponds to 1 naut mile at the earth's surface; hence a rate of 0.1 deg/hr is 6 naut miles/hr. Present operational SINS systems have much better performance than this.

The Navy first contracted in 1948 for the development of a combination gyrocompass and stable vertical after progress in the FEBE airborne system led to speculations concerning potential marine applications. The resulting MAST (Marine Stable Element) system was tested at sea in 1953. As a result of early component and subsystem test, and as a greater understanding of inertial navigation evolved from early systems analysis, MIT suggested to the Office of Naval Research the development of SINS, which was begun in March 1951, completed in mid-1954, and tested at sea in late 1954 and early 1955. The great success of the SINS development at MIT led the Navy to install inertial navigation systems (by Autonetics or Sperry) in Polaris submarines. Ultimately, SINS systems were installed in aircraft carriers and in range ships for the Atlantic and Pacific Missile Ranges. Advanced SINS work is also being carried out at Nortronics, MIT, and Honeywell, among others. Increases in accuracy and reliability, together with considerable reductions in size, weight, and power, have been achieved. Improved gyros and techniques of monitoring platform gyros with a redundant gyro are among the important improvements.

Missiles

The most comprehensive inertial guidance developments in the United States are for our ballistic missile systems. Table 2 shows the organizations with primary responsibilities for the guidance portion of the major missile programs. Inertial guidance systems have also been developed for cruise missiles, which are more like aircraft systems than ballistic missile systems. Examples are shown in Table 3.

Missile guidance systems have grown smaller and more accurate with each new development. Some of the advances of the past year or two are as follows:

- 1) Microelectronic computers have brought about significant reductions in size, weight, and power while at the same time improving reliability.
- 2) New platform concepts have been shown to be technically practical; it is now possible to float the stable platform as a ball inside a concentric sphere, thus permitting high- g operations. This means that a ballistic missile can be guided throughout its trajectory, including re-entry, instead of just during the boost phase as is now done.
- 3) Flotation of conventional gimballed systems to increase g capabilities without degradation in performance has been demonstrated.
- 4) With guidance using techniques (2) or (3), it is possible to maneuver the re-entry vehicle to improve penetrability without loss in accuracy.
- 5) Stars have been tracked during boost. Considerably improved measurements of the sky light background during day and night have been recorded by high-flying aircraft, balloons, and ballistic missiles.
- 6) Considerable technical progress has been made in gyrocompassing techniques and accuracies for both mobile and fixed missiles.

There has been a steady reduction in the size of an inertial measurement unit (IMU); systems in advanced development

are the size of a softball. Further size reduction appears to be beyond the foreseeable state-of-the-art.

Production costs, of secondary concern to accuracy and reliability in long-range ballistic missile systems, is of major importance in short range missiles. The d.c. Automet guidance system in the Army's Lance missile is one attractive approach toward lower guidance system costs. The Navy is developing a low-cost inertial system for ship-to-shore fire-support missiles of tactical ranges.

Space Guidance and Control

The drive for space exploration has kindled men's minds and has brought on a host of challenging new engineering problems, not the least of which are injection guidance, in-space navigation³⁷ and attitude control,³⁸ and re-entry guidance.³⁹ NASA has entered the guidance field as an important customer as well as a significant house for guidance research, particularly in the guidance laboratory at the Marshall Space Flight Center. This laboratory, headed by Walter Haeussermann, has evolved from the original Penemunde organization and must be credited with many original developments, from the first V-2 guidance system to the Saturn guidance system currently being tested. In the early 1950's, this group began seriously to look at orbital and space guidance problems and concepts.⁴⁰

Attitude control of orbital spacecraft is closely related to the guidance problem, particularly in some instrumentation approaches. Among organizations that have contributed significant work in this area are Honeywell, United Aircraft Corporate Systems, and Reeves Instruments. Passive attitude control for satellites in the form of gravity gradient stabilization has made considerable progress in the past year, particularly as a result of space experiments performed by the Applied Physics Laboratory, Johns Hopkins University. Gravity gradient control of communication and navigation satellites should be common in the future.

Reliability is of great importance in all guidance systems, particularly in space missions. In order to obtain the required reliability in booster guidance, it is necessary that a few standard boosters have the ability to inject into suitable trajectories a variety of spacecraft with widely different space missions. It is only through such repetitive use that the reliability can be improved to the very high level required.

Space not only introduces new problems into the guidance and navigation art, it also introduces new solutions to age-old problems. The following quotation is taken from Vice Admiral John T. Hayward's paper⁴¹ on space technology and world navigation:

Space is a great stepping stone for improving man's standard of living and for broadening his knowledge of the world in which we live. I feel that the Navy satellite navigation system⁴⁶ is one of the great fundamental strides upward in the direction of utilizing space to improve the world as we know it.

NASA, in the Apollo program and the DoD in the Standardized Space Guidance System (SSGS), are approaching the space guidance problem from widely divergent views. Each concept appears to be sound for the particular objectives being sought. The SSGS concept views the space mission as a whole from launch to touch-down. The same basic system, or portions thereof, is used for boost, injection, in-orbit guidance and attitude control, retrothrust, and re-entry. The Apollo concept, on the other hand, views the space mission as a series of separate phases^{42, 43}; the Saturn booster, the Command and Service Module, and the Lunar Excursion Module, each will have its own independent guidance and control system, with appropriate interfaces between phases.

Space guidance and control hardware is largely an outgrowth of missile technology. The Titan III uses a modified version of the Titan II guidance system. The Saturn booster uses hardware which is a logical extension of Jupiter and Pershing, designed by the Marshall Space Flight Center and

Table 3 Inertial guidance systems

Program	Organization
Hound Dog	Autonetics
Snark	Nortronics
Navaho	Autonetics
Mace	AC Spark Plug
Regulus	AC Spark Plug
Rascal	Bell Aerosystems

manufactured by Bendix. The Apollo IMU, designed by MIT and manufactured by AC Spark Plug, is an outgrowth of the Polaris Mk II system, which was designed by MIT and manufactured by General Electric. However, the space environment is vastly different from the missile environment. Operating times are much longer; accelerations and vibrations run the gamut from nothing in space to very high levels during boost and re-entry; power requirements, reliability, and temperature control problems are generally severe in both missile and space applications. Strap-down inertial systems appear to be practical in orbit and may be useful for boost and re-entry as well.⁴⁴ Microelectronics are just as important in space as in missiles, if not more so. Long-time space operations put a premium on electromagnetic sensors such as infrared horizon scanners, star trackers, and ground tracking data links.^{45, 46} Pure inertial systems, without external aids, are not practical for space applications except for very short missions.

Information Exchange

One of the important recent advances made in the guidance and control field is in communications among scientists and engineers interested in this field and actively engaged in work in the field. The AIAA, for example, has convened two specialists meetings in guidance and control. The first of these, under ARS sponsorship prior to formation of the AIAA, was in August 1961 at Stanford; the second was in August 1963 at MIT.

Among the difficulties in communication among guidance and control specialists are considerations of military security and company proprietary rights. The high security classification placed by the military departments on inertial guidance in the late 1940's and most of the 1950's began to be relaxed about 1956 or so. As a result, the unclassified literature has expanded tremendously in the past five to seven years (in addition to previously indicated references, see Refs. 47-62). There are at least seven books devoted exclusively to inertial guidance and inertial sensors (see Refs. 63 and 64 in addition to those previously indicated), and numerous others contain one or more chapters that discuss the subject in an excellent manner.

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